The Eulerian and Lagrangian Analysis of an Unsteady Wake Behind a Circular Cylinder

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Objective: To determine the advantages and disadvantages of Eulerian methods and a Lagrangian method used to identify coherent structures in the wake of a circular cylinder. Particular emphasis is placed on the near wake.

Overview: Despite a good qualitative understanding of coherent structures (vortices), no distinct quantitative definition of their boundaries exists. Several methods of identifying coherent structures in fluid flows have been defined and most are Eulerian in nature. Recent work using a Lagrangian method of analysis (Green, Haller, Rowley, 2007) has proven to be successful in identifying distinct coherent structures. The flow around a circular cylinder was chosen in this case because it is a well understood and well documented flow. It is known that vortices are shed in an unsteady manner from a cylinder at a Reynolds number of approximately 47.

Eulerian Criterion

Three Distinct Eulerian methods of Analysis were implemented: The Q criterion first proposed by Hunt, Wray, and Moin, the Δ criterion of Chong, Perry, and Cantwell and the vorticity.

Q Criterion

Q: a scalar value that measures the dominance of the rate of rotation (Ω) over the rate of strain (S)

Q = S · Ω

Δ criterion

Δ: scalar value that finds regions where the eigenvalues (k) of the gradient of the velocity are complex (local spiraling)

Δ = det(Vₐ)

Coherent Structures are regions in the flow where Q > 0

Vorticity

Regions of high magnitude vorticity indicate coherent structures

ω = ∇ × u

Characteristics of Eulerian Methods

-Relies on instantaneous velocity fields
-Can be used to implement and compute
-Galilean invariant and therefore not frame independent
-Requires the user to set some threshold value to define the boundaries of the coherent structures

Vorticity Field and LCS superimposed to analyze the vortices in the near wake region

LCS are superimposed onto the Q criterion results and the vorticity analysis results

Discussion and Conclusion

Q criterion shows location of vortex cores and boundaries

D criterion shows relative strength of vortices
-nLCS curls in direction of rotation of vortices
-nLCS arcs between neighboring vortices
-vortices in the near wake region

LCS are plotted in White

Near Cylinder Analysis

Eulerian and Lagrangian Comparison

-Positive vortex fully developed and has been shed
-ΔLCS divides region of positive vorticity into two regions: a vortex that is developing and a vortex that has just been shed
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References


Experimental Set-Up and Numerical Simulation

- 3D Cross Flow Numerically Simulated with NASA OVERFLOW software
- 2D plane from the mid cylinder plane was taken for analysis
- Eulerian analysis completed in Matlab and the Lagrangian analysis was completed using the program by Green, Rowley, Smits (2003)

Experiment Set-Up

Cylinder Diameter: 5 ft
Free Stream Velocity: 120 ft/sec
Temperature: 60°F
Reynolds Number: 150
Strohlield Number: 1

Lagranian Analysis

-Defines coherent structures boundaries objectively

Based on particle trajectories to determine regions of separation and attraction

Large measure of separation indicate two regions in the flow with distinctly different properties

Finite Time Lyapunov Exponent (FTLE) scalar value that measures separation among trajectories

Maximizing ridges in the FTLE field are Lagrangian Coherent Structures (LCS)

To identify LCS a threshold value is applied to filter out lower value ridges

Integrating forward in time produces the pLCS and the pLCS integrates backward in time produces the nLCS and the nLCS indicates regions where flow diverges in real time, nLCS indicates regions where flow converges in real time